

LESSON PLANS

Title – 3000-year-old stories of the square root of 2	
Lesson plan to be implemented in Math class	
Aim	<p>The main goal of this lesson plan is:</p> <ul style="list-style-type: none"> - to travel back two and three thousand years in time, when our ancient ancestors had identified the irrationality of the diagonal of a square and, in fact, had estimated its length- to build a small robot from everyday items; - to know the definitions of the square root, the properties of square roots, and irrational numbers
Students age targeted	8 th grade students / 14-15 years old
Estimated time	60 - 70 minutes
Topics covered	<ul style="list-style-type: none"> ● STEAM ● Real-world application ● powers, roots, and ratios
Facility/ Equipment	<ul style="list-style-type: none"> ● Classroom ● Internet access ● infographic, ● comic, ● papers A4, ● ruler ● Handout 1 – comic, Handout 2 - infographic
Tools/ Materials	<ul style="list-style-type: none"> ● Article: A 3,800-year journey from classroom to classroom https://news.yale.edu/2016/04/11/3800-year-journey-classroom-classroom ● Video: The 5 best proofs that the square root of 2 is irrational https://www.youtube.com/watch?v=zEXcsZo4hOQ&ab_channel=TippingPointMath ● Square Root: MathWorld - Wolfram Web Resource https://mathworld.wolfram.com/SquareRoot.html ● Geogebra application www.geogebra.org ● Construction of the square roots of 1,2,3... Wheel of Theodorus https://www.geogebra.org/m/BwKQyu69

<p>Development of activities</p>	<p>Activity 1: The definitions of square root and irrational number</p> <p>What is the definition of square root? What is the definition of irrational number? Students discuss the above question, watch to the video regarding “The 5 best proofs that the square root of 2 is irrational” and answer the teacher’s questions (MathWorld - Wolfram Web Resource about square root).</p> <p>Activity 2: Construction of the square roots of 1, 2, 3... from the unit [1]</p> <p>Students in groups observe the construction of the square roots of 1, 2, 3..., using the GeoGebra app “Wheel of Theodorus”. They try to construct their own artistic version using Geogebra or other tool. Other constructions are in this Students’ Art Work.</p> <p>Activity 3: The activity with the four A4 papers!</p> <p>What is the ratio of the dimensions of different types of printing papers... A4, A3, A2, A1, A0? Students in groups have four A4 papers and try to answer the following questions: Which is the width and the height of the A4 What is the ratio width/height of the A4? Make an A3 from two A4 What is the ratio width/height of the A3? What is the ratio width/height of the A2? What is your conclusion?</p> <p>Activity 4: Discover - Explore - Enjoy Art...</p> <p>Can you spot the math hidden in these works of art, of the following links? National Museum of American History, Johnson Crockett https://americanhistory.si.edu/collections/search?edan_local=1&edan_q=square%20root%20Johnson%20Crockett&edan_fg%5B0%5D=online_visual_material%3Atrue</p> <p>'Riu-Kiu-C' by Vasarely http://www.op-art.co.uk/op-art-gallery/victor-vasarely/riu-kiu-c</p>
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HANDOUT 1: Comic poster

3000-year-old stories of the square root²

M₁

The Babylonian Tablet: A 3,800-year journey from classroom to classroom

How can we double a square?

Plato's 'Meno' (82b9-85c3) Double the area of a square Socrates' dialectical method!

If I have a square with each side being 1, how can I double the area?

That's simple! Double each of the sides...

Come on! Focus!

Oops! I was wrong. If I double each side, I will have a square where each side will be two. Therefore, the area will be quadrupled, not doubled...

Let's draw a shape that may be helpful

It is certain that the dimension of the square in question is bigger than 1 and smaller than 2. Here it is! I drew the squares: one has a dimension of 1 and an area of 1, and the other has a dimension of 2 and an area of 4.

Do you know about the diagonal of a square?

The diagonal separates the square into two identical right triangles.

Each one has half the area of the original.

1.41421
35623730
950488016
887242096980
78569671875376
48073176679737990
75247846210703885038
753432764157275501384
6230912297024924836055
8507372126441214970999

$\sqrt{2}$ is irrational

Is it easier now to draw a square having an area of 2, that is, half the area of the first square?

Yes, it's easier!

What about the diagonal?

I haven't calculated it exactly... I'd say about 1.41?

M₂ the definition of square root

The square root of a positive number 'θ' is:

- a positive number 'a', which, when raised to the power of 2 yields 'θ', or
- the positive solution of the equation: $x^2 = \theta$ (this equation has 2 solutions: $x = \sqrt{\theta}$ & $x = -\sqrt{\theta}$)

M₃ properties

- $\sqrt{a} \geq 0, a \geq 0$
- $(\sqrt{a})^2 = a, a \geq 0$
- $\sqrt{a^2} = |a|, a \in \mathbb{R}$
- $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}, a, b \geq 0$
- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}, a \geq 0, b > 0$

M₄ the definition of irrational number

Irrational numbers are numbers that cannot be written as a fraction of integers. An irrational number can be written as a decimal number with infinite, non-periodical digits.

[The 5 best proofs that the square root of 2 is irrational](#)

[Square Root \[MathWorld - Wolfram Web Resource\]](#)

HANDOUT 2: Infographic

Powers - Roots - Ratios in our World!

S₁ powers & roots in Nature...

How to construct the square roots of 1, 2, 3... from the unit

Wheel of Theodorus
Student Art Work The Greek Version

E₁ in Architecture...

According to Sebastiano Serlio (1475-1554), an ideal pedestal is one with its dimensions yielding the diagonal ratio which is... $\frac{\text{height}}{\text{width}} = \sqrt{2}$

Tutte l'opere d' architettura et prospetiva

E₂ paper in our everyday life

What is the ratio of the dimensions of different types of printing papers... A5, A4, A3, A2, A1, A0? A5 is half the size of A4, A4 is half the size of A3 and so on...

$\frac{x}{\psi} = \frac{\psi}{2x}$

T without words, with coding blocks!

Half-baked nested squares and square roots

A in Art...

Square Roots of 1,2,3
Square Roots to 16
Squares of 1, 2, 3, 4
and Square Roots to 8

National Museum of American History

'Riu-Kiu-C' (Vasarely)

S T E A M problems!

1. A mathematician of the x-th century A.D. (x>2) said: "The square root of the year I was born is exactly the same as my current age". How old was he and when was he born?
2. Can you escape from Dr Schrödinger's lair? You must output to the display the numbers 2, 10, and 14. The control panel has only 3 buttons that change the display: +5, +7, and square root. Watch the first 2' of the video!
3. Can you calculate the length of the colored shapes in the tangram (the side length = 1)?

E₃ the photographic diaphragm

The lens opening gradation to light exposure, forms a decreasing geometric sequence. What is the ratio?

$f/1.4, f/2 = \frac{f/1.4}{\sqrt{2}}, f/2, f/2.8 = \frac{f/2}{\sqrt{2}}, f/2.8, f/4 = \frac{f/2.8}{\sqrt{2}}, f/4$
 $f/5.6 = \frac{f/4}{\sqrt{2}}$
 $f/5.6, f/8 = \frac{f/5.6}{\sqrt{2}}, f/8, f/11 = \frac{f/8}{\sqrt{2}}, f/11, f/16 = \frac{f/11}{\sqrt{2}}, f/16$

S₂ in paving...

"Tessellations in Aqueleia's Mosaics"

root (noun): from Old Norse, from the Indo-European root *wrad-* "branch, root". A plant can't exist without its roots, so metaphorically speaking *root* means "basis" or "source". When you say that the cube root of 8 is 2, 2 is the basic number that has to be raised up, like a plant from its roots, until it becomes 8. Also, when we solve an equation, we are looking for the solutions that we assume to be "rooted" in the equation; each solution is called a root of the equation. *The Words of Mathematics*